$\sigma$ = molecular interaction parameter, (2)
$\sigma_i$ = standard deviation of $i$
$\tau$ = $tD_e/L^2$
$\Omega$ = molecular potential energy parameter, (2)

**Subscripts**

1 = species one
2 = species two
$e$ = effective
$k$ = Knudsen
$s$ = surface

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# Temperature Gradients in Turbulent Gas Streams:

## Investigation of the Limiting Value of Total Prandtl Number

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**Measurements of the total viscosity and total conductivity were made as a function of position and Reynolds number for the flow of air between two parallel plates with a separation of approximately 0.7 in. The temperature of the upper plate was 130.0°F. and that of the lower plate 70.0°F. The investigations were carried out at Reynolds numbers from 40,000 to 100,000 and were in good agreement with earlier data obtained at Reynolds numbers up to 40,000. The results obtained indicate little change in the total Prandtl number with increasing Reynolds number from the value of the molecular Prandtl number.**

An understanding of the interrelation of momentum and energy transport in turbulent flow is a matter of engineering interest. A great deal of effort has been directed to the measurement of momentum transfer from knowledge of local values of velocity and shear. More limited investigations have been carried out in the field of thermal transfer. The experimental techniques in the field of thermal transfer are often simpler and may permit a more accurate evaluation of such transport phenomena than can be obtained readily from the measurements of momentum transfer. The solution of the equations representing the energy balance in a turbulent stream can be readily determined for a variety of conditions (*15, 27*), provided local values of thermal transport are available.

Reynolds (*26*) and Prandtl (*25*) indicated the basic characteristics of convective thermal transfer when treated as directly analogous to momentum transfer. The work of von Karman (*17*) outlined the principal relationships associated with the concepts of eddy viscosity and eddy conductivity which afford a useful, although empirical, tool in the evaluation of the effect of turbulence upon the transfer of momentum and energy. Boelter and co-workers (*3*) extended the Reynolds analogy (*26*) and the concepts of von Karman (*17*), and utilized such approaches to predict the thermal transfer to fluids flowing in conduits. More recent studies (*8, 13, 22, 24, 27*) have determined more fully the effect of Reynolds number and position in the flow channel upon the values of the eddy viscosity and the eddy conductivity for a steady, nearly uniform flow of air between parallel plates.

### ANALYTICAL RELATIONS

von Karman (*17*) defined eddy viscosity for steady, uniform flow between parallel plates as:

$$\epsilon_m = \frac{\tau g}{\sigma \dfrac{du}{dy}} - \nu \tag{1}$$

The total viscosity was defined as the sum of the kinematic viscosity and eddy viscosity:

$$\underline{\epsilon}_m = \epsilon_m + \nu = \frac{\tau g}{\sigma \dfrac{du}{dy}} \tag{2}$$

For uniform flow between parallel plates, the shear in Equations (1) and (2) is defined by the following expression:

$$\tau = \frac{y_0}{2}\left(-\frac{dP}{dx}\right)\left(1 - 2\frac{y}{y_0}\right) \tag{3}$$

It should be noted that Equations (1) and (2) cannot be used directly to evaluate values of eddy and total viscosity at the axis of the flow since both the shear and the velocity gradient are zero at this point. However, by application of L'Hospital's rule to Equation (3), the following expression was obtained, which permits a direct evaluation of eddy and total viscosity at the center of the channel:

$$\epsilon_m = \underline{\epsilon}_m - \nu = \frac{g\dfrac{dP}{dx}}{\sigma \dfrac{d^2u}{dy^2}} - \nu \tag{4}$$

The values of eddy and total viscosity can also be estimated analytically (*4, 27*) from available generalizations (*1, 16*) of the velocity profile for flow between parallel plates. It should be pointed out that this approach yields zero values of total viscosity at the center of the conduit. However, experimental evidence (*22, 24, 27*) indicates that this situation is by no means true. For this reason it does not appear desirable to utilize these analytical expressions to establish the total viscosity near the center of the channel.

The eddy and total conductivities have been defined in the following way:

$$\epsilon_c = \frac{\dot{q}}{C_p \sigma}\frac{dy}{dt} - \mathrm{K} \tag{5}$$

$$\underline{\epsilon}_c = \epsilon_c + \mathrm{K} = \frac{\dot{q}}{C_p \sigma}\frac{dy}{dt} \tag{6}$$

All the variables involved in Equations (5) and (6) may be measured directly under steady, uniform conditions. However, the thermal flux obtained by conventional calorimetric techniques only provides information concerning the flux at the upper boundary. To obtain the local fluxes, the effect of viscous dissipation must be taken into account. The contribution of the thermal flux due to viscous dissipation may be approximated by (*29*):

$$\dot{q}_j = \int_{y_0}^{y} \eta\, \Phi\, dy = \int_{y_0}^{y} \tau \frac{\partial u}{\partial y} dy = \frac{1}{2}\left(-\frac{dP}{dx}\right)\int_{y_0}^{y} (y_0 - 2y)\frac{du}{dy} dy \tag{7}$$

Hence, the corrected local overall thermal flux becomes:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_a + \dot{q}_j \tag{8}$$

where $\dot{q}_a$ is the thermal flux at the upper wall obtained directly from calorimetric measurements (*28, 29*).

In the treatment of the relationship between momentum and thermal transfers, the Prandtl number is often useful. The molecular Prandtl number is defined as the ratio of the kinematic viscosity to the thermometric conductivity. The ratio of the total viscosity to the total conductivity has been defined (*9*) as the total Prandtl number:

$$\underline{N}_{Pr} = \frac{\underline{\epsilon}_m}{\underline{\epsilon}_c} = \frac{C_p \tau g}{\dot{\mathbf{q}}}\frac{dt}{du} \tag{9}$$

Correspondingly, the eddy Prandtl number may be defined as:

$$N_{Pr\epsilon} = \frac{\epsilon_m}{\epsilon_c} = \underline{N}_{Pr}\left(1 + \frac{\mathrm{K}}{\epsilon_c}\right) - \frac{\nu}{\epsilon_c} \tag{10}$$

On the basis of earlier experimental information (*9, 13, 24*), it appears that for a fluid with a molecular Prandtl number corresponding to that of air, the ratio of total viscosity to total conductivity, hereafter called the *total Prandtl number*, does not approach unity at Reynolds numbers up to 40,000. However, no experimental information was available concerning the values of total viscosity and total conductivity at higher Reynolds numbers. It was of interest to establish the trend of the total Prandtl number as the Reynolds number was still further increased above 40,000. The purpose of the present investigation was to establish values of eddy and total viscosity and conductivity at Reynolds number between 40,000 and 100,000 for a steady, nearly uniform air stream flowing between parallel plates.

## EXPERIMENTAL METHODS

The equipment employed in these investigations has been described in detail elsewhere (*7, 8*). It consisted of two parallel, copper plates between which air was circulated under steady, nearly uniform conditions. The channel was approximately 13 ft. in length, 12 in. in width, and 0.7 in. in height. The temperatures of the upper and lower surfaces of the duct were maintained at constant but different values by circulating oil above and below the parallel plates. Under such conditions a temperature gradient normal to the direction of flow was imposed upon the air stream. The local velocity was established by means of a 0.008 in. pitot tube and a hot-wire anemometer mounted on traversing equipment, whereas the gross flow rate was measured by the use of a Venturi meter in the supply duct. The actual vertical position of the traversing pitot tube and hot-wire anemometer was determined by means of a small cathetometer mounted upon the traversing equipment.

The hot-wire anemometer consisted of a 1 mil platinum wire 0.4 in. in length. The constant resistance method was applied in the local velocity measurements with the anemometer. The anemometer was calibrated against pitot tube measurements near the center of the channel. In applying the constant resistance technique, the platinum wire was held at an optimum operating temperature of about 50°F. above that of the air stream. Such an approach decreased changes in the resistance characteristics of the wire due to aging. Corresponding values of velocity were obtained with both the hot-wire anemometer and the pitot tube. These data were used to establish the calibration of the hot-wire anemometer. The hot-wire anemometer was also used as a resistance thermometer. After making a velocity measurement the resistance of the wire at the stream temperature was established. Measurements with the traversing equipment indicated that the

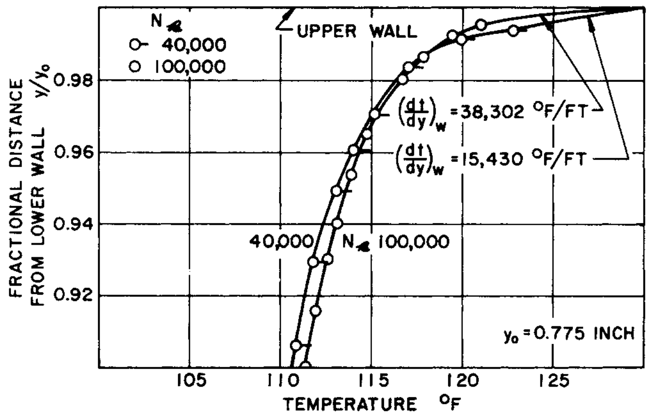


**Fig. 1. Temperature distribution near upper wall.**

hot wire could be located within 0.003 in. relative to the copper plates. The behavior of the anemometer near the wall has been described elsewhere (8).

The pressure gradients in the flowing stream were ascertained from piezometer bars used in conjunction with kerosene-in-glass manometers observed with a cathetometer (7). In addition, the pressure was measured at the position of the traversing equipment which was 88.6 in. downstream of the approach section. Thermocouples were provided in the upper and lower copper plates to establish the temperature at the boundary of the air stream. The thermal flux normal to the air stream was determined by the use of two calorimeters located in the upper plate, one near the downstream end and the other near the middle of the flow channel (7). The copper plates were parallel within 0.005 in. throughout the working section. The width of the channel did not vary by more than 0.010 in. throughout the length.

A knowledge of the relationship between the pressure and the distance along the channel was used to determine the uniformity of the flow. Under uniform velocity for a fluid at constant specific volume and viscosity, a linear relationship should exist between the pressure and the position along the channel. As would be expected for a compressible fluid, the pressure gradient at the downstream end of the channel was slightly larger than at the entrance. However, the weight rate of flow of air was uniform as long as there were no leaks of air to or from the channel through the side blocks. Consequently, the Reynolds number of the flow was uniform throughout the channel. In any event, the deviation from uniform flow was sufficiently small as not to impair the analysis of the thermal transfer measurements.

In the present investigation, most of the measurements were made with the upper and lower plate maintained at 130°F. and 70°F., respectively, whereas the entering bulk air temperature was maintained at 100°F. Effort was directed to the refinement of the apparatus in order to permit the experimental measurements to be of as high an accuracy as was feasible. The temperatures were measured with resistance thermometers, which were compared with a similar instrument calibrated by the National Bureau of Standards, with an error of not more than 0.05°F., and the thermal flux at the upper wall was established with a standard error of estimate of not more than 1%. The probable error associated with the velocity and shear measurements was estimated to be less than 1%. The details of velocity, temperature, energy, and pressure measurements followed closely the methods described earlier (8) and it does not appear necessary to describe further the details of the experimental techniques.

Table 1 records the properties of air which were employed in the analysis of this work. Most of these data were based upon those selected by Keenan and Kaye (18). The primary uncertainty associated with the application of these values resulted from the presence of a small amount of moisture in the air used. The influence of composition upon specific volume of the air-water system in the gas phase at atmospheric pressure was estimated on the assumption that the phase was an ideal solution. The values of specific volume so obtained compared well with available measured values for gaseous mixtures of air and water. The viscosity of the air-water mixture was established from the theoretical considerations suggested by Chapman and Cowling (5). The details of these calculations are available elsewhere (23).

Near the wall the error in establishing the position of the wire exceeded the uncertainty in the measurements of the velocity and temperature of the air. It was found helpful to establish the behavior near the boundary from the knowledge of the limiting velocity and temperature gradient at the wall. The velocity gradient at the wall was evaluated from

$$\left(\frac{du}{dy}\right)_w = \frac{g\tau_0}{\sigma\nu_w} \tag{11}$$

Equation (11) can also be written in terms of the pressure gradient as

$$\left(\frac{du}{dy}\right)_w = \frac{gy_0}{2\sigma\nu_w}\left(-\frac{dP}{dx}\right) \tag{12}$$

Correspondingly, the temperature gradient at the wall was given by

$$\left(\frac{dt}{dy}\right)_w = -\frac{\dot{q}_w}{k} \tag{13}$$

From these limiting gradients and a few experimental data in the vicinity of the walls, the velocity and temperature profiles near the boundaries were established more accurately than would otherwise be possible. Figure 1 illustrates the variation of temperature with position near the wall for two Reynolds numbers. The significant effect of Reynolds number upon the temperature distribution near the wall is evident.

Table 1. Properties of Dry Air at Atmospheric Pressure

| Property | Reference | Units | Temperature, °F. 70 | 100 | 130 |
|---|---|---|---|---|---|
| isobaric heat capacity | (11, 18, 19) | B.t.u./(lb.)(°F.) | 0.2403 | 0.2406 | 0.2409 |
| kinematic viscosity × $10^4$ | (18, 19, 21, 24) | sq.ft./sec. | 1.64 | 1.81 | 1.98 |
| molecular Prandtl number | | | 0.713 | 0.710 | 0.707 |
| specific volume | (14, 18, 24) | cu.ft./lb. | 13.35 | 14.11 | 14.86 |
| thermal conductivity × $10^6$ | (24) | B.t.u./(sec.)(ft.)(°F.) | 4.14 | 4.34 | 4.54 |
| thermometric conductivity × $10^4$ | (24) | sq.ft./sec. | 2.30 | 2.54 | 2.80 |
| viscosity × $10^7$ | (18, 19, 21, 24) | (lb.)(sec.)/sq.ft. | 3.82 | 3.98 | 4.19 |

Table 2. Local Values of Total Viscosity*

| Fractional distance from lower wall | Reynolds Number 17,000 | 40,000 | 60,000 | 80,000 | 100,000 |
|---|---|---|---|---|---|
| 0.0 | $0.164 \times 10^{-3}$ | $0.164 \times 10^{-3}$ | $0.164 \times 10^{-3}$ | $0.164 \times 10^{-3}$ | $0.164 \times 10^{-3}$ |
| 0.02 | 0.74 | 1.27 | 1.76 | 2.35 | 3.37 |
| 0.05 | 1.68 | 2.97 | 4.78 | 6.60 | 8.97 |
| 0.10 | 2.92 | 5.26 | 8.32 | 10.73 | 13.20 |
| 0.20 | 4.40 | 8.00 | 11.09 | 14.20 | 16.95 |
| 0.30 | 4.76 | 7.89 | 11.05 | 13.77 | 16.83 |
| 0.40 | 4.04 | 6.43 | 8.90 | 11.58 | 14.63 |
| 0.50 | 3.62 | 5.65 | 8.15 | 10.81 | 13.80 |
| 0.60 | 4.20 | 6.51 | 8.96 | 11.75 | 14.78 |
| 0.70 | 4.90 | 8.27 | 10.85 | 14.15 | 17.05 |
| 0.80 | 4.62 | 8.22 | 10.82 | 14.68 | 17.17 |
| 0.90 | 2.99 | 5.71 | 8.29 | 11.03 | 13.93 |
| 0.95 | 1.42 | 3.43 | 5.12 | 7.20 | 10.61 |
| 0.98 | 0.58 | 1.55 | 2.24 | 3.01 | 5.20 |
| 1.00 | 0.198 | 0.198 | 0.198 | 0.198 | 0.198 |

* Total viscosity expressed in sq.ft. per sec.

In the measurement of the temperature of a moving stream by means of a stationary wire, it is necessary to take into account the rise in temperature of the fluid immediately surrounding the wire as a result of the viscous dissipation and pressure distribution in the vicinity of the wire. For this purpose, it was found convenient to utilize the recovery factor concept to determine the actual temperature of the flowing air. This factor (*21, 28*) is defined as

$$\xi = \frac{T_0 - T}{T_s - T} = \frac{T_0 - T}{u^2/(2gC_p)} \tag{14}$$

Earlier studies with air (*9, 10, 22*) yielded recovery factors varying from 0.62 to approximately 0.69. In the present investigation a recovery factor of 0.66 was employed (*21*). This corresponds to a maximum temperature deviation of 0.22°F. at a Reynolds number of 40,000 and a maximum temperature deviation of 1.30°F. at a Reynolds number of 100,000 at the center of the channel. The temperature measurements have been corrected for such effects.

## RESULTS

A series of experimental measurements of the velocity and temperature distribution in steady, nearly uniform flow was made at Reynolds numbers between 40,000 and 100,000. The Reynolds number was evaluated in the following fashion:

$$N_{Re} = \frac{2\, y_0\, U}{(\nu)_{y/y_0=0.5}} = \frac{2}{(\nu)_{y/y_0=0.5}} \int_0^{y_0} u\,dy = \frac{2\, y_0\, \dot{m}}{\eta\, g\, A} \tag{15}$$

It is noted from Table 1 that the variation of kinematic viscosity with temperature in the range of conditions encountered in the present investigation is virtually linear. For this reason the value of viscosity at the center plane of the channel was employed in Equation (15) instead of using a space-averaged value. The last equality involving the weight rate of flow was used in the actual calculation of the Reynolds number.

Because the qualitative aspects of these measurements are similar to those found earlier (*13, 22*), there does not appear to be any justification for graphical presentation of the velocity and temperature distributions. Measurements were made in a regular sequence in order to establish the general trend of the velocity and temperature with position. Also for each test, several measurements were made in which the position was chosen at random in order to determine the reproducibility of the measurements. These random measurements did not yield greater standard errors of estimate from the velocity and temperature profiles than the data obtained in a regular sequence of positions.

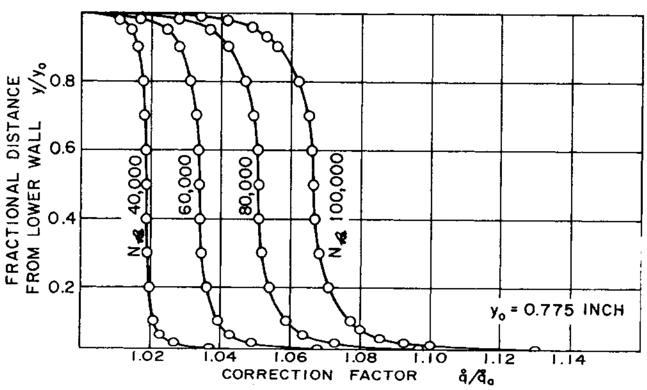

Fig. 2. Effect of Reynolds number and position upon viscous dissipation.

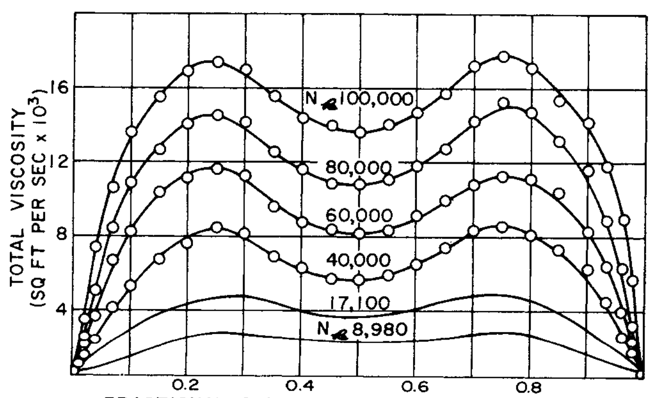

Fig. 3. Experimental values of total viscosity.

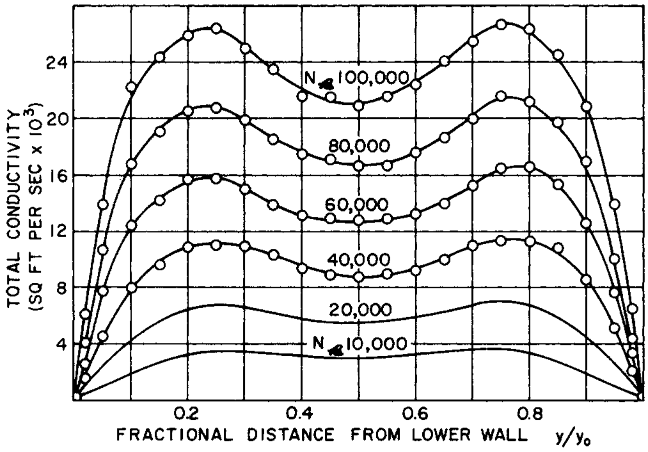


**Fig. 4. Experimental values of total conductivity.**

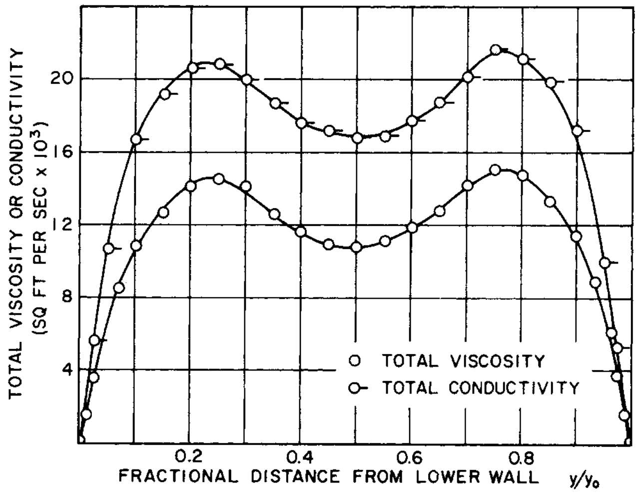


**Fig. 5. Comparison of total viscosity and total conductivity for a Reynolds number of 80,000.**

Utilizing the measurements of velocity and shear as a function of position across the channel, the measured values of thermal flux were corrected for the effect of viscous dissipation. This correction factor has been defined (*29*) as the ratio of the corrected value of heat flux to the uncorrected value. In carrying out these calculations, the values of thermal flux $\dot{q}_a$ determined from calorimetric measurements (*8, 28*) were employed for the evaluation of the term $\dot{q}$, as indicated by Equation (8). The quantity $\dot{q}_j$ was established by application of Equation (7) to the measured values of pressure gradient and velocity gradient.

The extent of the correction for viscous dissipation is illustrated in Figure 2 as a function of position in the channel for several Reynolds numbers. It is evident that the correction is zero at the top of the channel and attains a maximum at the bottom. The contribution due to viscous dissipation is much greater in the boundary flows than in the central part of the stream. Furthermore, the correction factor increases rapidly with increase in Reynolds number.

From the information concerning the flow conditions and the available velocity distribution and pressure difference, values of the total and eddy viscosity were calculated by application of Equations (1) and (2). Such calculations established, from the current data, the total and eddy viscosity as a function of position for Reynolds numbers between 40,000 and 100,000. Comparison was made with the earlier data (*24*) and good agreement was obtained between the two sets of point values of total viscosity for air flowing between parallel plates at Reynolds numbers of approximately 40,000. The current and older data (*24*) established the total viscosity as a function of position in the stream. Experimental values of the total viscosity for several different Reynolds numbers are depicted in Figure 3. The data shown in Figure 3 were similar in form to those obtained earlier (13, 24). The minima at the center of the channel become more pronounced at the higher Reynolds number.

Table 2 sets forth values of the total viscosity as a function of position in the channel for different Reynolds numbers. The smoothed values of the total viscosity recorded in Table 2 should not involve a probable error greater than 3% of the tabulated value.

Correspondingly, point values of the total and eddy conductivity were calculated from the experimental temperature distribution and thermal flux by application of Equations (5) and (6). The results of such calculations for several Reynolds numbers are indicated in Figure 4 as a function of position in the channel. These data were corrected for viscous dissipation as set forth in Equations (7) and (8). The behavior of the total conductivity appears to be analogous to that for total viscosity. Smoothed values of total conductivity as a function of position in the

TABLE 3. LOCAL VALUES OF TOTAL CONDUCTIVITY*

| Fractional distance from lower wall | Reynolds Number | | | | | |
|---|---|---|---|---|---|---|
| | 10,000 | 20,000 | 40,000 | 60,000 | 80,000 | 100,000 |
| 0.0 | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ | $0.23 \times 10^{-3}$ |
| 0.02 | 0.34 | 0.93 | 1.76 | 2.75 | 3.95 | 6.15 |
| 0.05 | 0.89 | 2.40 | 4.57 | 7.73 | 10.51 | 14.18 |
| 0.10 | 1.84 | 4.35 | 8.03 | 12.38 | 16.81 | 21.57 |
| 0.20 | 3.24 | 6.46 | 10.87 | 15.70 | 20.60 | 25.86 |
| 0.30 | 3.45 | 6.60 | 11.03 | 14.99 | 19.95 | 25.27 |
| 0.40 | 3.17 | 5.81 | 9.44 | 13.17 | 17.65 | 22.12 |
| 0.50 | 3.05 | 5.52 | 8.82 | 12.70 | 16.76 | 21.16 |
| 0.60 | 3.30 | 5.89 | 9.42 | 13.28 | 17.62 | 22.67 |
| 0.70 | 3.61 | 6.78 | 11.00 | 15.11 | 20.18 | 25.67 |
| 0.80 | 3.31 | 6.78 | 11.28 | 16.47 | 21.08 | 26.10 |
| 0.90 | 1.89 | 4.57 | 8.72 | 12.73 | 16.58 | 20.83 |
| 0.95 | 0.97 | 2.55 | 5.01 | 7.85 | 9.70 | 13.40 |
| 0.98 | 0.41 | 1.12 | 2.00 | 3.51 | 4.25 | 6.43 |
| 1.00 | 0.28 | 0.28 | 0.28 | 0.28 | 0.28 | 0.28 |

* Total conductivity expressed in sq.ft. per sec.

TABLE 4. LOCAL VALUES OF TOTAL PRANDTL NUMBER

| Fractional distance from lower wall | Reynolds Number 10,000 | 20,000 | 30,000 | 40,000 | 60,000 | 80,000 | 100,000 |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.713 | 0.713 | 0.713 | 0.713 | 0.713 | 0.713 | 0.713 |
| 0.1 | 0.759 | 0.762 | 0.736 | 0.655 | 0.672 | 0.638 | 0.612 |
| 0.2 | 0.808 | 0.733 | 0.710 | 0.736 | 0.706 | 0.689 | 0.656 |
| 0.3 | 0.803 | 0.787 | 0.740 | 0.715 | 0.737 | 0.690 | 0.666 |
| 0.4 | 0.776 | 0.745 | 0.711 | 0.681 | 0.676 | 0.656 | 0.661 |
| 0.5 | 0.749 | 0.707 | 0.682 | 0.641 | 0.642 | 0.645 | 0.652 |
| 0.6 | 0.742 | 0.778 | 0.740 | 0.691 | 0.675 | 0.667 | 0.652 |
| 0.7 | 0.784 | 0.802 | 0.757 | 0.752 | 0.718 | 0.701 | 0.664 |
| 0.8 | 0.817 | 0.741 | 0.698 | 0.729 | 0.657 | 0.696 | 0.658 |
| 0.9 | 0.789 | 0.744 | 0.727 | 0.655 | 0.651 | 0.665 | 0.669 |
| 1.0 | 0.707 | 0.707 | 0.707 | 0.707 | 0.707 | 0.707 | 0.707 |
| Space avg.* | 0.768 | 0.747 | 0.720 | 0.698 | 0.687 | 0.679 | 0.665 |
| flow avg.† | | | | 0.696 | 0.682 | 0.672 | 0.655 |

* Space average defined by $\frac{1}{y_0}\int_0^{y_0} N_{Pr}\,dy$.

† Flow average defined by $\frac{1}{y_0 U}\int_0^{y_0} N_{Pr}\,u\,dy$.

channel are recorded in Table 3 for several Reynolds numbers. The details of the experimental data for total viscosity and conductivity are available elsewhere (*6*).

It should be pointed out that the values of total viscosity and total conductivity are sensitive to the velocity and temperature distributions in the channel. As is shown in Figures 3 and 4, a small lack of symmetry about the horizontal axis of the channel persisted in the experimental values of total viscosity and total conductivity. It is not believed that the asymmetry of these data resulted from changes in conditions during measurements but rather as a result of a lack of symmetry of the molecular properties in the flowing stream as a result of the transverse temperature gradient. Figure 5 shows a comparison between the values of total viscosity and total conductivity at a Reynolds number of 80,000.

The influence of Reynolds number upon the relative conductivity in the boundary flow near the upper wall is indicated in Figure 6. Similar behavior was found near the lower wall. As indicated earlier, the flow is nearly uniform in all respects throughout the flow channel at distances greater than 40 in. from the entrance. The behavior predicted from the Reynolds analogy is shown as a dotted curve. The deviation from the analogy is many times the experimental uncertainty.

The effect of viscous dissipation upon the value of total conductivity is demonstrated in Figure 7 for a Reynolds number of 100,000. The dotted curve represents the values of total conductivity uncorrected for viscous dissipation. The symmetry of the data involving the corrected values of total conductivity is markedly better than for the uncorrected data.

The values of total Prandtl number were calculated from the information concerning total viscosity and total conductivity. The results are presented in Table 4 as a function of position and Reynolds number. The space-average values of the total Prandtl number were also evaluated as a function of Reynolds number. The full curve of Figure 8 depicts the variation in the space-average value of total Prandtl number with the reciprocal Reynolds number. Total Prandtl numbers at several positions in the channel are also included as experimental points in Figure 8. The results indicate that the total Prandtl number is relatively insensitive to the Reynolds number of the flow and shows no trend toward a value of unity as the Reynolds number is increased. The values of the total Prandtl number are nearly equal to the molecular Prandtl number even at high Reynolds numbers.

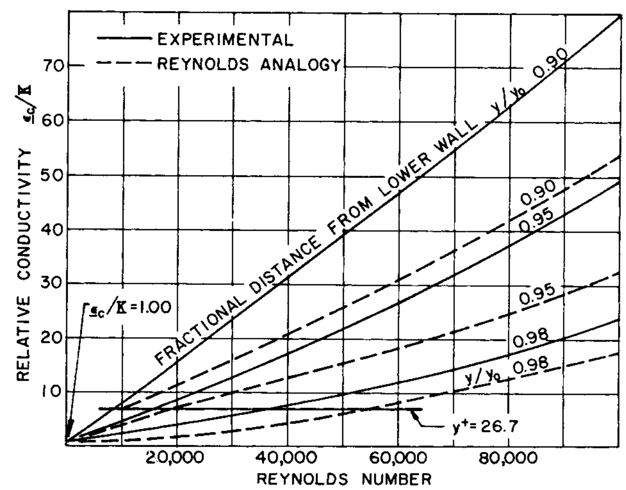


Fig. 6. Comparison of Reynolds analogy with experimental measurements near the upper wall.

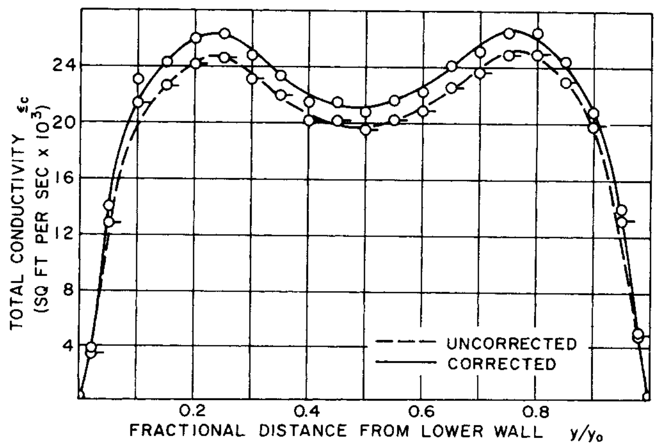


Fig. 7. Effect of viscous dissipation upon total conductivity for a Reynolds number of 100,000.

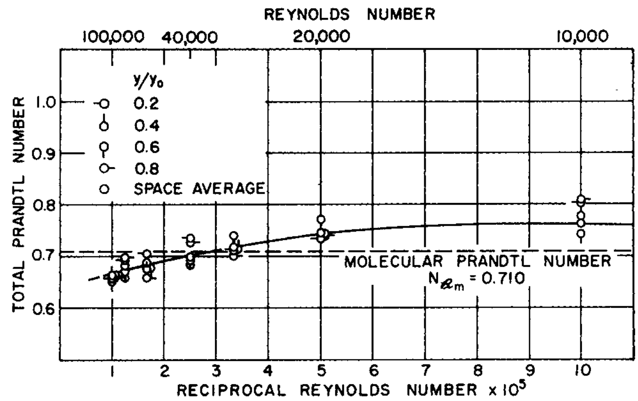


**Fig. 8. Experimental values of total Prandtl number as a function of reciprocal Reynolds number.**

Until further investigations are carried out involving fluids with molecular Prandtl numbers which differ markedly from unity, it remains to be ascertained if the trend indicated in this investigation for air persists for fluids which possess Prandtl numbers markedly greater or smaller than unity. If such trends are encountered for other fluids, it becomes a relatively simple matter to predict local thermal transfer in turbulent flow for situations where the local momentum transport is known. The trends indicated by this current investigation show that the molecular properties of the fluid exert a controlling influence upon the transport even under rather highly turbulent conditions.


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## NOTATION

$A$ = area, sq. ft.
$C_p$ = isobaric heat capacity, B.t.u./(lb.) (°F.)
$d$ = differential operator
$g$ = acceleration due to gravity, ft./sec.$^2$
$k$ = thermal conductivity, B.t.u./(sec.) (ft.) (°F.)
$\overset{\circ}{m}$ = weight rate of air flow, lb./sec.
$\overline{N_{Pr}}$ = total Prandtl number
$N_{Pr\epsilon}$ = eddy Prandtl number
$N_{Re}$ = Reynolds number
$\underset{\circ}{P}$ = pressure, lb./sq. ft.
$\underset{\circ}{q}$ = thermal flux, B.t.u./(sec.) (sq. ft.)
$q_j$ = local thermal flux due to viscous dissipation, B.t.u./(sec.) (sq. ft.)
$t$ = temperature, °F.
$T$ = absolute temperature, °R.
$T_0$ = temperature at surface of the wire, °R.
$T_s$ = temperature at the stagnation point, °R.
$U$ = gross velocity, ft./sec. = $(1)/(y_0) \int_0^{y_0} u dy$
$u$ = local time-average velocity, ft./sec.
$x$ = downstream distance along axis of stream, ft.
$y$ = distance normal to axis of stream measured from lower plate, ft.
$y^+$ = distance parameter = $yd/\nu \ (\tau/\sigma)^{1/2}$
$y_d$ = distance from nearer wall, ft.

**Greek Letters**

$\epsilon_c$ = eddy conductivity, sq. ft./sec.
$\underline{\epsilon}_c$ = total conductivity, sq. ft./sec.
$\epsilon_m$ = eddy viscosity, sq. ft./sec.
$\underline{\epsilon}_m$ = total viscosity, sq. ft./sec.
$\eta$ = absolute viscosity, (lb.) (sec.)/sq. ft.
$K$ = thermometric conductivity, sq. ft./sec., $k/\sigma C_p$
$\nu$ = kinematic viscosity, sq. ft./sec.
$\xi$ = recovery factor
$\sigma$ = specific weight, lb./cu. ft.
$\tau$ = shear, lb./sq. ft.
$\tau_0$ = shear at the wall, lb./sq. ft.
$\Phi$ = dissipation function, sec.$^{-2}$

**Subscripts**

$a$ = at upper plate
$w$ = wall
$x$ = in the $x$ direction
$y$ = in the $y$ direction

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